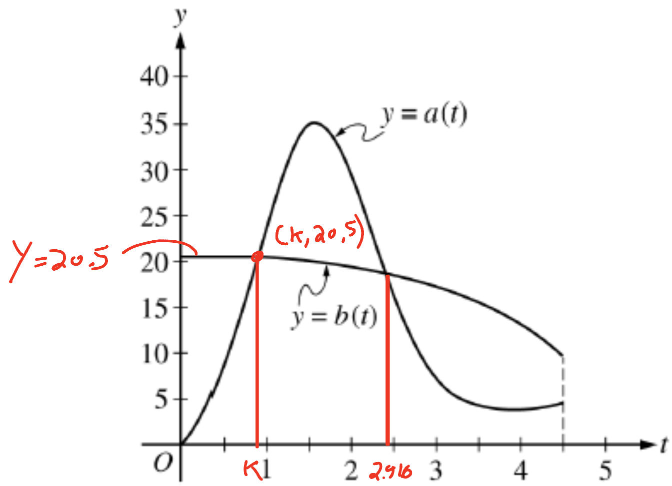
Period 3, May 9, 2025

~~Cross section~~
~~Seperation of variables~~
~~FRQ 2017 #2~~
~~Related rates lamp/shapes~~
~~U sub integration~~
Linear approx
~~Disc/washer~~

A GRAPHING CALCULATOR IS REQUIRED FOR THIS QUESTION.




During the time interval $0 \leq t \leq 4.5$ hours, water flows into tank *A* at a rate of $a(t) = (2t-5) + 5e^{2\sin t}$ liters per hour. During the same time interval, water flows into tank *B* at a rate of $b(t)$ liters per hour. Both tanks are empty at time $t = 0$. The graphs of $y = a(t)$ and $y = b(t)$, shown in the figure above, intersect at $t = k$ and $t = 2.416$.

(a) How much water will be in tank *A* at time $t = 4.5$ ?

$$V = \int_0^{4.5} [(2T-5) + 5e^{2\sin t}]\,dx$$

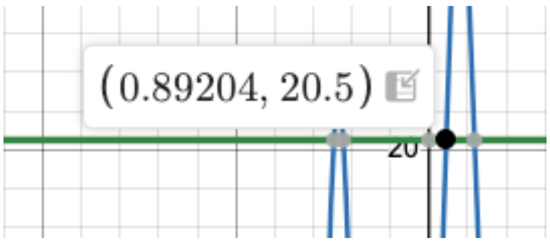
$$\int_0^{4.5} \left[ (2t-5) + 5e^{2\sin t} \right] dt = 66.5321282934$$

66.532 Liters

(b) During the time interval $0 \le t \le k$ hours, water flows into tank *B* at a constant rate of 20.5 liters per hour. What is the difference between the amount of water in tank *A* and the amount of water in tank *B* at time $t = k$?

$20.5 = (2T-5) + 5e^{2\sin T}$

Find $T$




$$\int_0^{0.89204} \left[(2t-5) + 5e^{2\sin t}\right] dt$$

= 7.68762870238

$\boxed{K = 0.89204}$

Tank $B = \int_0^{0.89204} 20.5\, dT$

$20.5T \Big]_0^{0.89204}$

$= 18.28682$

Tank $A = \int_0^{0.89204} [(2T-5) + 5e^{2\sin T}]\, dT$

$= 7.68762870238$

$18.28682 - 7.6876287$

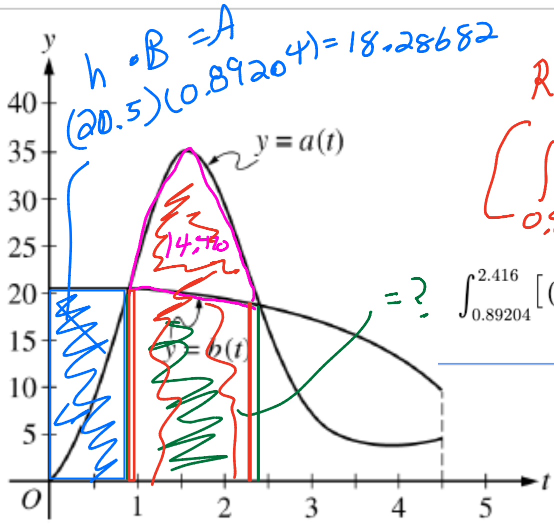
10.599 Liters

(c) The area of the region bounded by the graphs of $y = a(t)$ and $y = b(t)$ for $k \le t \le 2.416$ is 14.470.

How much water is in tank *B* at time $t = 2.416$?

$h \cdot B = A$

$(20.5)(0.89204) = 18.28682$




$= ?$

Red − Pink = green

$\left[\int_{0.89204}^{2.416} a(T)\right] - 14.470 = \int_{0.89204}^{2.416} b(T)\, dT$

$$\int_{0.89204}^{2.416} \left[(2t-5) + 5e^{2\sin t}\right] dt$$

= 44.497056407

$44.497056407 - 14.470 = \int_{0.89204}^{2.416} b(T)\, dT$

30.027051

+ 18.28682

= 48.313876

(d) During the time interval $2.7 \le t \le 4.5$ hours, the rate at which water flows into tank *B* is modeled by $w(t) = 21 - \frac{30t}{(t-8)^2}$ liters per hour. Is the difference $w(t) - a(t)$ increasing or decreasing at time $t = 3.5$ ? Show the work that leads to your answer.

$w'(T) - a'(T)$ = increasing $+$

decreasing $-$

4

$$\frac{d}{dt}\left[(2t-5)+5e^{2\sin t}\right] = -2.6430274906$$

5

$t = 3.5$

$-10$ 10

6

$$\frac{d}{dt}\left[21 - \frac{30t}{(t-8)^2}\right] = -3.78600823045$$

7

$-3.7860082 - (-2.643027) =$

$-3.786 + 2.643 =$ negative

decreasing

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$$\cancel{\frac{dx}{y}} \cdot \frac{dy}{\cancel{dx}} = \frac{5x\cancel{y} \cdot dx}{\cancel{y}}$$

$$\int \frac{1}{y}\,dy = \int 5x\,dx$$

$$\ln_e y = \frac{5}{2}x^2 + C$$

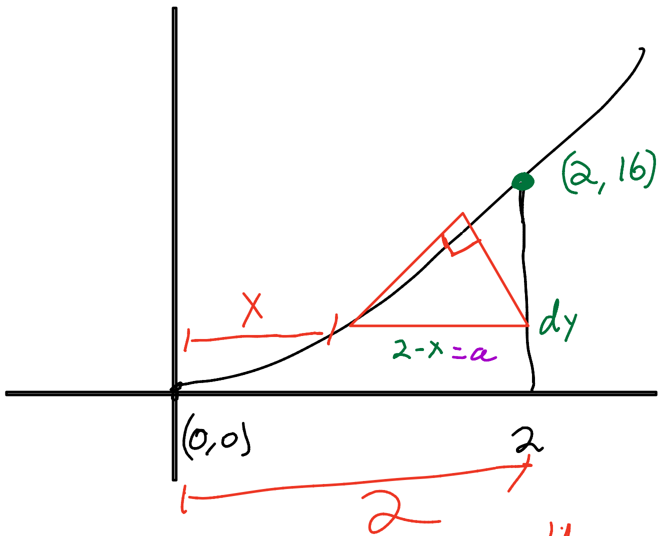
$$e^{\frac{5}{2}x^2 + c} = y$$

$$e^{\frac{5}{2}x^2} \cdot e^{c} = y$$

$$e^{\frac{5}{2}x^2} \cdot C_1 = y$$

$$\log_b a = c \iff b^c = a$$

$$\ln_e a = c \iff e^c = a$$




$$y = 4x^2 \Rightarrow x = \frac{\sqrt{y}}{2}$$

$$\frac{1}{2}\int_0^{16} (2-x)\left(\frac{a^2\sqrt{2}}{8}\right) dy$$

$$\frac{1}{2}\int_0^{16} [(2-x)\left(\frac{(2-x)^2 \cdot \sqrt{2}}{8}\right) dy$$

$$\frac{1}{2}\int_0^{16} [(2-\frac{\sqrt{y}}{2})\left(\frac{(2-\frac{\sqrt{y}}{2})^2 \cdot \sqrt{2}}{8}\right) dy$$

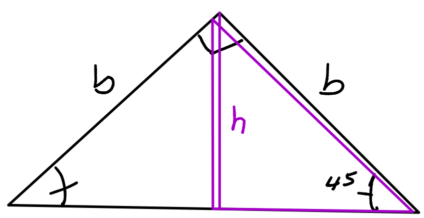
$$\frac{\sqrt{2}}{16}\int_0^{16} \left(2-\frac{\sqrt{y}}{2}\right)^3 dy$$

$$\sin 45^\circ = \frac{h}{b}$$

$$b \cdot \frac{\sqrt{2}}{2} = \frac{h}{\cancel{b}} \cdot \cancel{b}$$

$$\frac{\sqrt{2}}{2} \cdot b = h$$

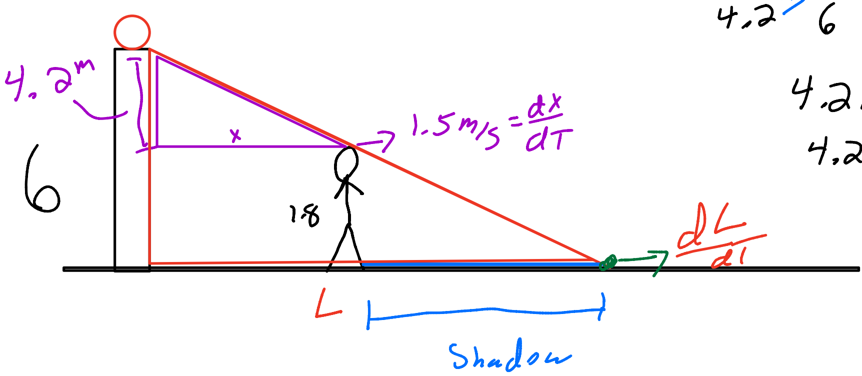
$$\frac{\sqrt{2}}{2} \cdot \frac{a^2}{4} = h$$




$$a = 2\sqrt{b}$$

$$\frac{a^2}{4} = b$$

A 1.8-meter tall man walks away from a 6.0-meter lamp post at the rate of 1.5 m/s. The light at the top of the post casts a shadow in front of the man. How fast is the "head" of his shadow moving along the ground?




$$\frac{x}{4.2} = \frac{L}{6}$$

$$4.2L = 6x$$

$$4.2\frac{dL}{dT} = 6\frac{dx}{dT}$$

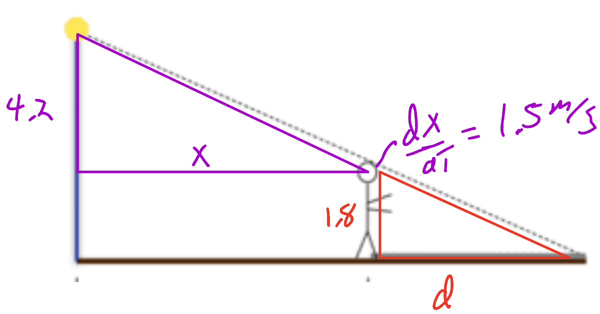
$$4.2\frac{dL}{dT} = 6(1.5)$$

$$\frac{dL}{dT} = \frac{9}{4.2}$$

$$\frac{dL}{dT} = 2.142857 \text{ m/s}$$

How fast is the length. Of the shadow changing.

Find $\frac{dd}{dT}$




$$\frac{4.2}{x} = \frac{1.8}{d}$$

$$1.8x = 4.2d$$

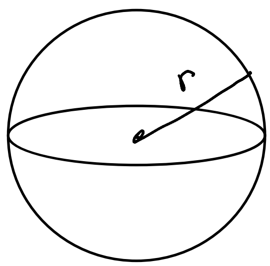
$$1.8\frac{dx}{dT} = 4.2\frac{dd}{dT}$$

$$\frac{(1.8)(1.5)}{4.2} = \frac{dd}{dT}$$

$$0.642857 \text{ m/s}$$

$2.142857 - 1.5 = 0.642857$

Shadow Head Rate — walking Rate — Shadow Length Rate




$$\frac{dV}{dT} = -4\ cm^3/s \quad \text{deflating}$$

Find How Fast The Radius is changing when $r = 7cm$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dT} = \frac{4}{\cancel{3}}\pi \cdot \cancel{3} r^2 \frac{dr}{dT}$$

$$\frac{dV}{dT} = -4\ cm^3/s = 4\pi r^2 \frac{dr}{dT}$$

$$-4\ cm^3/s = 4\pi(7cm)^2 \frac{dr}{dT}$$

$$\frac{-4\ cm^{\cancel{3}\,1}/s}{196\pi\, \cancel{cm^2}} = \frac{\cancel{196\pi\, cm^2}}{\cancel{196\pi\, cm^2}} \frac{dr}{dT}$$

$$\frac{-4}{196\pi}\ cm/s = \frac{dr}{dT}$$

$$\frac{-1}{49\pi}\ cm/s = \frac{dr}{dT}$$

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$$\int \frac{x}{\sqrt{3x+4}}\,dx \qquad u = 3x+4 \Rightarrow x = \frac{u-4}{3}$$

$$du = 3dx$$

$$\frac{du}{3} = dx$$

$$\int \frac{x}{\sqrt{u}} \cdot \frac{du}{3}$$

$$\int \frac{\frac{u-4}{3}}{\frac{\sqrt{u}}{1}} \frac{du}{3} = \int \frac{u-4}{3} \cdot \frac{1}{\sqrt{u}} \cdot \frac{du}{3} = \frac{1}{9}\int \frac{u-4}{\sqrt{u}}\,du$$

$$\frac{1}{9}\int \left[\frac{u}{\sqrt{u}} - \frac{4}{\sqrt{u}}\right] du$$

$$\int_{2}^{6} (x-4)^5 (e^{x-4}) dx$$

$U = x-4 \Rightarrow du = dx$

$-2 = 2-4$

$2 = 6-4$

$$\int_{-2}^{2} u^5 e^u du$$

$$\frac{1}{9}\int (u^{\frac{1}{2}} - 4u^{-\frac{1}{2}}) du$$

$$\frac{1}{9}\left[\frac{2}{3}u^{\frac{1}{2}+1=\frac{3}{2}} - 4 \cdot 2 \cdot u^{-\frac{1}{2}+1=\frac{1}{2}}\right]$$

$$\frac{1}{9}\left[\frac{2}{3}u^{3/2} - 8u^{\frac{1}{2}}\right] + C$$

$$\frac{1}{9}\left[\frac{2}{3}(3x+4)^{3/2} - 8\sqrt{3x+4}\right] + C$$

$3x^2 = T$ $\quad$ $6x\,dx = dT$

$$\frac{d}{dx}\left[\int_{5}^{3x^2} e^{\sin 4T^3} (\cos 6T)(T^2 - 6T)\, dT\right]$$

$$e^{\sin 4(3x^2)} (\cos 6(3x^2)) \left((3x^2)^2 - 6(3x^2)\right) 6x$$

$g(x) = F^{-1}(x)$

$F(x) = 3x^3 - 2x^2 + x - 4$

**Theorem 4.80. Derivative of Inverse Functions.** *Given an invertible function $f(x)$, the derivative of its inverse function $f^{-1}(x)$ evaluated at $x = a$ is:*

$$\left[f^{-1}\right]'(a) = \frac{1}{f'\left[f^{-1}(a)\right]}$$

Find $g'(14)$

$$g'(14) = \frac{1}{F'(g(14))} = \frac{1}{F'(2)} = \frac{1}{29}$$

Find
$g(14) = a$
$F(a) = 14$

$F(2) = 14$
$g(14) = 2$

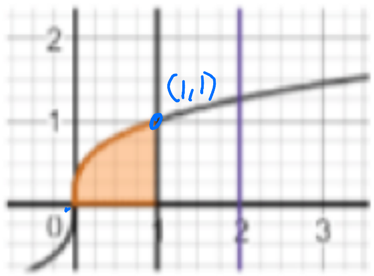
$14 = 3x^3 - 2x^2 + x - 4$

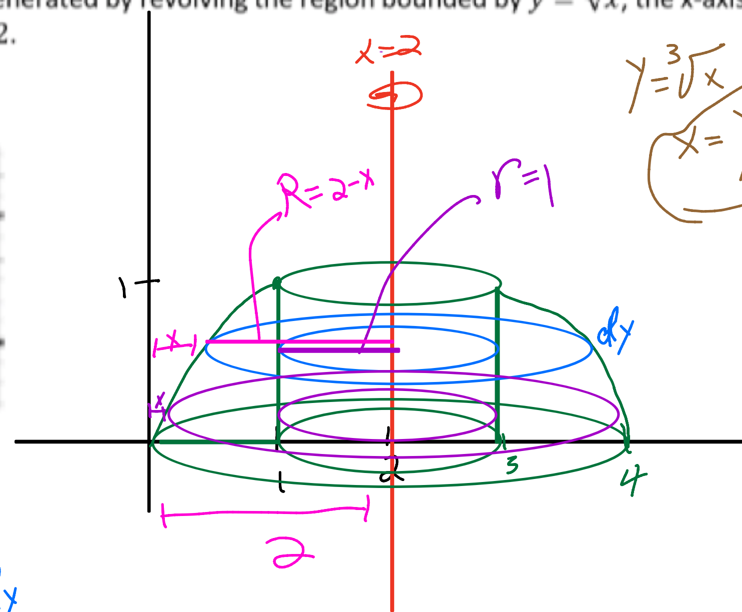
$F(x) = 3x^3 - 2x^2 + x - 4$
$F'(x) = 9x^2 - 4x + 1$
$F'(2) = 9(2)^2 - 4(2) + 1$
$36 - 8 + 1 = 29$

| x | y |
|---|---|
| 0 | $-4 = F(0)$ |
| $\pm 1$ | $-2 = F(1)$ |
| $\pm 2$ | $14$ $\quad F(2) = 3(2)^3 - 2(2)^2 + 2 - 4$ |
| $\pm 3$ | |
| $-1$ | $-10 = F(-1)$ |

7.

Find the volume of the solid generated by revolving the region bounded by $y = \sqrt[3]{x}$, the x-axis, and the line $x = 1$ about the line $x = 2$.




$y = \sqrt[3]{x}$

$x = y^3$

$\pi\int_0^1 (R^2 - r^2)\,dy$

$\pi\int_0^1 ((2-x)^2 - (1)^2)\,dy$

$\pi\int_0^1 [(2-y^3)^2 - 1]\,dy = \pi\int_0^1 [4 - 4y^3 + y^6 - 1]\,dy = \int_0^1 [3 - 4y^3 + y^6]\,dy$

$3y - y^4 + \frac{1}{7}y^7 + C \Big|_0^1$

$\pi\left[3(1) - 1^4 + \frac{1}{7}(1)^7 - \left[\cancel{3(0) - 0^4 + \frac{1}{7}(0)^7}\right]\right]$

$3 - 1 + \frac{1}{7} = 2\frac{1}{7}\pi$

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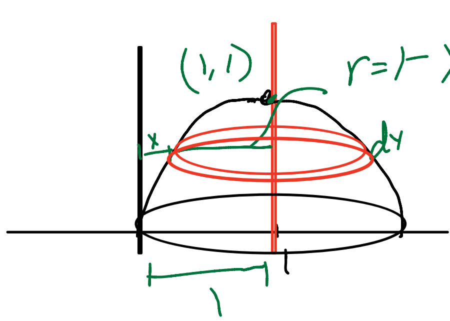


$\int_0^1 \pi r^2\,dy$

$\pi\int_0^1 (1-x)^2\,dy$

$\pi\int_0^1 (1-y^3)^2\,dy$

$\pi\int_0^1 (1 - 2y^3 + y^6)\,dy$

$\pi(y - y^3 + \frac{1}{7}y^7)\Big|_0^1$

$1 - 1 + \frac{1}{7}$

$\frac{1}{7}\pi$

**1.** $f(x) = 3x\,e^{2x-10}$ at $x = 5$

Create a Line

m and Point

Use Linear approx

To Find $\boxed{x = 5.1}$ $\Delta x = 0.1$

$F(5.1) \approx 18.3$

$F(5) = 3 \cdot 5 e^{2 \cdot 5 - 10} = 15 \cdot e^{0} = 15 \cdot 1$

$= 15$

$(5, 15)$

$F'(5) = 33 = m$

$m = F'(x) = 3 \cdot e^{2x-10} + 3x \cdot e^{2x-10} \cdot 2$

$F'(5) = 3 \cdot e^{10-10} + 3 \cdot 5 e^{10-10} \cdot 2$

$= 3 + 30 = 33$

$y - 15 = 33(x - 5) + 15$

$+15$

$\rightarrow y = 33(x-5) + 15$

$33(5.1 - 5) + 15$

$33(0.1) + 15$

$3.3 + 15$

$18.3$

$\frac{dy}{dx} = \frac{\text{change in } y}{\text{change in } x} =$

$\frac{dy}{dx} = 3e^{2(5)-10} + 3(5) \cdot e^{2(5)-10} \cdot 2$

$(0.1) \cdot \frac{dy}{0.1} = 33(0.1)$

$dy = 3.3$

$\Delta x = 0.1$ $(5, 15)$

$(5.1, 18.3)$ $3.3$

$y = ce^{kT}$

$k = +$ growth

$k = -$ decay

$y = kx$

$x \cdot y = \frac{k}{\cancel{x}} \cdot \cancel{x}$